Dust-acoustic modes in self-gravitating plasmas with dust size distributions

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Using a kinetic description, dust-acoustic waves are considered for dusty plasmas containing, besides the electrons and ions, dust particles with continuous mass (size) distributions. For broad size spectra, self-gravitational effects cannot be neglected anymore because in the competition between electromagnetic and gravitational forces, the scale tips over towards gravitation for the heavier dust grains. Self-gravitational effects are clearly interwoven with the grain size distribution and here the effects of different power-law size distributions on the propagation, damping, and instability of low-frequency waves are discussed.

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I. INTRODUCTION

A consistent modeling of wave processes in dusty plasmas runs into interesting difficulties that do not occur in standard plasma theory. Besides dealing with particles that can have fluctuating charges, the dust grains come in all sizes, in an almost continuous range going from macromolecules to rock fragments and asteroids. While many papers have tried to account in some way for charge fluctuations, as reviewed in a recent book [1], comparatively little has been done about other facets of dust grains, like their distribution in mass and size.

Many authors have considered charged dust as a limited number of discrete species (discussed by Verheest [1]). Rather than describing the dust as a number of discrete species, other attempts have been made to treat the charged dust density as a continuous distribution over a limited size range, by using a decreasing power law. This results in a dust distribution damping even in the fluid theory [2]. In a different vein Brattli *et al.* [3] considered dust size distributions from a kinetic point of view, and found that Landau damping dominates at short wavelengths, whereas for larger wavelengths attenuation due to charge variations becomes more important. Changes in dispersion relations have only been given for some of the better-known dusty plasma modes such as dust-acoustic waves.

However, when dealing with a broad size distribution, self-gravitation could become important for the heavier dust grains, may be even comparable in magnitude to electromagnetic forces. This implies that self-gravitational interactions should be included into the analysis. When self-gravitation is incorporated into a fluid description stable and unstable modifications of the dust-acoustic mode are found [4–9].

In the present paper we also focus on the analogues of the dust-acoustic modes [10–12] in self-gravitating plasmas, but the modifications due to the different size distributions are studied within a kinetic description. We thus generalize our previous kinetic approach [13] to include a size distribution.

The plan of our paper is as follows. In Sec. II we recall some of the basic results of kinetic theory and discuss different size distributions, Sec. III is devoted to the analysis of the dispersion of dust-acoustic waves for different size distributions, and Sec. IV summarizes our results.

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II. KINETIC DISPERSION RELATION FOR A DUSTY SELF-GRAVITATING PLASMA

Rather than repeat the standard kinetic equations, we will straightaway make use of the very general dispersion relation for electrostatic modes in a self-gravitating plasma, which demonstrates the coupling between the plasma and gravitational branches of oscillations from a kinetic point of view [13] as well as in the fluid approach, namely,

$$\varepsilon(\omega,k) \equiv \varepsilon_p(\omega,k) + \frac{K^2(\omega,k)}{\varepsilon_J(\omega,k)} = 0, \tag{1}$$

which involves a plasma dielectric constant

$$\varepsilon_p = 1 + \frac{1}{\varepsilon_0 k^2} \sum_\alpha \frac{q_\alpha^2}{m_\alpha} I_\alpha, \tag{2}$$

its analogue for a self-gravitating neutral medium

$$\varepsilon_J = 1 - \frac{4\pi G}{k^2} \sum_\alpha m_\alpha I_\alpha, \tag{3}$$

and the coupling factor

$$K = \sqrt{\frac{4\pi G}{\varepsilon_0}} \frac{1}{k^2} \sum_\alpha q_\alpha I_\alpha, \tag{4}$$

with

$$I_\alpha = \int \frac{\mathbf{k}\cdot\nabla_v f_{\alpha 0}}{\omega - \mathbf{k}\cdot\mathbf{v}} d^3\mathbf{v}. \tag{5}$$

We have defined for the different species with label α standard notations q_α, m_α, and $f_{\alpha 0}$, for the charge, mass, and unperturbed distribution function, respectively.

We now assume that all plasma species are described by Maxwellian distribution functions

$$f_{\alpha 0}=\frac{n_{\alpha 0}}{(\pi v_{T\alpha}^2)^{3/2}}\exp\left(-\frac{v^2}{v_{T\alpha}^2}\right), \tag{6}$$

with $n_{\alpha 0}$ the equilibrium particle density, $v_{T\alpha}=\sqrt{2k_BT_\alpha/m_\alpha}$ the thermal velocity, T_α the temperature of particles of type α, and k_B the Boltzmann constant. Taking all this into account, the dispersion relation (1) can be written as

$$\varepsilon(\omega,k)\equiv 1+\sum_{\alpha=e,i,d}\frac{1+i\sqrt{\pi}z_\alpha W(z_\alpha)}{k^2\lambda_{D\alpha}^2}+\frac{\left(\sum_{\alpha=e,i,d}\frac{1+i\sqrt{\pi}z_\alpha W(z_\alpha)}{k^2\lambda_{D\alpha}\lambda_{J\alpha}}\right)^2}{1-\sum_{\alpha=e,i,d}\frac{1+i\sqrt{\pi}z_\alpha W(z_\alpha)}{k^2\lambda_{J\alpha}^2}}=0, \tag{7}$$

where

$$W(z)=\exp(-z^2)\left(1+\frac{2i}{\sqrt{\pi}}\int_0^z\exp(x^2)dx\right) \tag{8}$$

is the Kramp function [14] and $z_\alpha=\omega/kv_{T\alpha}$ a dimensionless frequency. The characteristic Debye and Jeans lengths are defined through $\lambda_{D\alpha}^2=\varepsilon_0 k_B T_\alpha/q_\alpha^2 n_{\alpha 0}$ and $\lambda_{J\alpha}^2=k_BT_\alpha/4\pi G m_\alpha^2 n_{\alpha 0}$.

To investigate the influence of dust mass distributions, we consider a self-gravitating plasma consisting of electrons (with subscript e), positively charged ions (subscript i), and negatively charged dust particles (subscript d). Since in real dusty plasmas dust grains can come in all sizes, in an almost continuous spectrum in a range $a_{\min}<a<a_{\max}$ (a is the dust particle size) we introduce the differential density, which usually goes as a power-law distribution like

$$n_d(a)da=N_0a_0^{\mu-1}a^{-\mu}da \quad (\mu>0), \tag{9}$$

such that the equilibrium density of all dust grains is given by

$$N_0=\int_{a_{\min}}^{a_{\max}}n_d(a)da. \tag{10}$$

Along with the total number density N_0, we have introduced a characteristic size a_0, which due to the normalization of the distribution function $n_d(a)$ can be defined through

$$\frac{a_0^{\mu-1}}{\mu-1}(a_{min}^{1-\mu}-a_{max}^{1-\mu})=1. \tag{11}$$

If the range of sizes is fairly wide, then $a_{\min}\ll a_{\max}$, and thus for $\mu>1$, $(a_0/a_{min})^{\mu-1}\simeq\mu-1$ holds.

Distributions of this sort have been observed in heliospheric dusty plasmas, with power law indices $\mu=4.6$ for the F ring of Saturn [15], $\mu=7$ [16] and $\mu=6$ [17] for the G ring, and $\mu=3.6$ in cometary environments [18]. Further references are given by Meuris *et al.* [6].

When the standard dusty plasma model is adhered to, mass and charge of a dust particle can be expressed as

$$m(a)=\frac{4}{3}\pi\rho a^3, \quad q(a)=4\pi\varepsilon_0 a\varphi_0, \tag{12}$$

where ρ is the mass density of the grain material and φ_0 the electric surface potential at equilibrium. Both characteristics are assumed to be constant and equal for all grains, to simplify the discussion.

If $\mu>2$, the equation of quasineutrality is given by

$$q_in_{i0}=en_{e0}+N_0q_1\frac{\mu-1}{\mu-2}, \tag{13}$$

where $q_1=q(a_{\min})=4\pi\varepsilon_0\varphi_0 a_{\min}$ is the charge of the smallest grains, q_i the charge of the ions, and n_{e0} and n_{i0} are the unperturbed electron and ion densities.

In the case of a continuous distribution (9), the discrete summation over different grain species in Eq. (7) is replaced by an integral, so that the dispersion law is now of the form

$$\left[1+\sum_{\alpha=e,i}\frac{1+i\sqrt{\pi}z_\alpha W(z_\alpha)}{k^2\lambda_{D\alpha}^2}+\int_{a_{\min}}^{a_{\max}}\frac{1+i\sqrt{\pi}z(a)W[z(a)]}{k^2\lambda_{Dd}^2(a)}da\right]\times\left[1-\int_{a_{\min}}^{a_{\max}}\frac{1+i\sqrt{\pi}z(a)W[z(a)]}{k^2\lambda_{Jd}^2(a)}da\right]+\left(\int_{a_{\min}}^{a_{\max}}\frac{1+i\sqrt{\pi}z(a)W[z(a)]}{k^2\lambda_{Dd}(a)\lambda_{Jd}(a)}da\right)^2=0, \tag{14}$$

with $\lambda_{Dd}^2(a)=\varepsilon_0k_BT/q^2(a)n_d(a)$ and $\lambda_{Jd}^2(a)=k_BT/4\pi m^2(a)n_d(a)$ denoting the analogues of the Debye and Jeans lengths, where the temperature of the dust particles is assumed to be the same for all species [viz. $T(a)=T\neq T_i$], $z_i=\omega/kV_{Ti}$, $z(a)=\omega/kv_T(a)$, and $v_T^2(a)=2k_BT/m(a)$.

Considering waves in a self-gravitational plasma, it is pointless to take the electron and ion gravitational interactions into account because of their relative smallness, so as always we have neglected all terms proportional to $\lambda_{Ji}^{-1}\sim m_i$ and *a fortiori* to $\lambda_{Je}^{-1}\sim m_e$.

III. DUST-ACOUSTIC MODES IN SELF-GRAVITATING PLASMAS

In the dust-acoustic regime $kv_T(a_{\min})\ll\omega\ll kv_{Ti}, kv_{Te}$, resulting in inequalities $z(a_{min})\gg 1$ and $z_e, z_i\ll 1$, one can use the asymptotic expansions for the Kramp function for small and large arguments [14]:

(i) $|z|\ll 1$,

$$W(z)=1+\frac{2iz}{\sqrt{\pi}}+\cdots;$$

(ii) $|z|\gg 1$,

$$W(z)=\frac{i}{\sqrt{\pi}z}\left(1+\frac{1}{2z^2}+\frac{3}{4z^4}+\cdots\right)+\exp(-z^2).$$

Inserting these into Eq. (14) gives us

$$\left[1+\sum_{\alpha=e,i}\frac{1+\dfrac{i\sqrt{\pi}\omega}{kv_{T\alpha}}}{k^2\lambda_{D\alpha}^2}-\frac{\Omega_p^2}{\omega^2}\left(1+\frac{\Delta_p}{\omega^2}\right)+i\omega C_p\right]\times\left[1+\frac{\Omega_J^2}{\omega^2}\left(1+\frac{\Delta_J}{\omega^2}\right)-i\omega C_J\right]+\left[\frac{\Omega_{pJ}^2}{\omega^2}\left(1+\frac{\Delta_{pJ}}{\omega^2}\right)-i\omega C_{pJ}\right]^2=0, \tag{15}$$

where the values of the effective dust plasma, Jeans, and hybrid frequencies are denoted by Ω_p, Ω_J, and Ω_{pJ},respectively are given through

$$\Omega_p^2=\int_{a_{\min}}^{a_{\max}}\frac{q^2(a)n_d(a)}{\varepsilon_0 m(a)}da=\int_{a_{\min}}^{a_{\max}}\tilde{\omega}_p^2(a)da, \tag{16}$$

$$\Omega_J^2=\int_{a_{\min}}^{a_{\max}}4\pi G m(a)n_d(a)da=\int_{a_{\min}}^{a_{\max}}\tilde{\omega}_J^2(a)da, \tag{17}$$

$$\Omega_{pJ}^2=\int_{a_{\min}}^{a_{\max}}\tilde{\omega}_p(a)\tilde{\omega}_J(a)da. \tag{18}$$

Equations (16) and (17) define $\tilde{\omega}_p^2(a)$ and $\tilde{\omega}_J^2(a)$, and the notations Δ_p, Δ_J and Δ_{pJ} were introduced to represent thermal corrections due to the plasma particles, namely,

$$\Delta_p=\frac{3k^2}{2\Omega_p^2}\int_{a_{\min}}^{a_{\max}}\tilde{\omega}_p^2(a)v_T^2(a)da, \tag{19}$$

$$\Delta_J=\frac{3k^2}{2\Omega_J^2}\int_{a_{\min}}^{a_{\max}}\tilde{\omega}_J^2(a)v_T^2(a)da, \tag{20}$$

$$\Delta_{pJ}=\frac{3k^2}{2\Omega_{pJ}^2}\int_{a_{\min}}^{a_{\max}}\tilde{\omega}_p(a)\tilde{\omega}_J(a)v_T^2(a)da. \tag{21}$$

Finally, the coefficients C_p, C_J, and C_{pJ} of the imaginary parts in Eq. (15) make its structure more compact and stand for

$$C_p=\frac{\sqrt{\pi}}{k^3}\int_{a_{\min}}^{a_{\max}}\frac{\exp[-\omega^2/k^2v_T^2(a)]}{v_T(a)\lambda_{Dd}^2(a)}da, \tag{22}$$

$$C_J=\frac{\sqrt{\pi}}{k^3}\int_{a_{\min}}^{a_{\max}}\frac{\exp[-\omega^2/k^2v_T^2(a)]}{v_T(a)\lambda_{Jd}^2(a)}da, \tag{23}$$

$$C_{pJ}=\frac{\sqrt{\pi}}{k^3}\int_{a_{\min}}^{a_{\max}}\frac{\exp[-\omega^2/k^2v_T^2(a)]}{v_T(a)\lambda_{Jd}(a)\lambda_{Dd}(a)}da. \tag{24}$$

The dispersion law (15) is clearly very complicated. All these dispersion relations can be solved numerically, of course, as given by Brattli *et al.* [3] for a dusty plasma without self-gravitation, but including charge fluctuations. However, it is instructive to find analytical solutions for different size spectra.

We first deal with the imaginary part of Eq. (15), and consider the integrals (22)–(24). These can be written as $C_\nu=Q\int_{a_{\min}}^{a_{\max}}a^\nu\exp[-\omega^2/k^2v_T^2(a)]da$, with Q constant, and computed in the same way with the help of the relation [19],

$$C_\nu=Q\int_{a_{\min}}^{a_{\max}}a^\nu\exp(-ba^3)da=-\frac{Q}{3}b^{-\nu+1/3}\Gamma\left(\frac{\nu+1}{3},ba^3\right)\Bigg|_{a_{\min}}^{a_{\max}}, \tag{25}$$

where $b=\omega^2/a^3k^2v_T^2(a)$ is independent of a and $\Gamma(p,x)$ denotes the incomplete gamma function

$$\Gamma(p,x)=\int_x^\infty\exp(-t)t^{p-1}dt. \tag{26}$$

The dust-acoustic regime corresponds to $kv_T(a)\ll\omega$ so that $ba^3=\omega^2/k^2v_T^2(a)\gg 1$. Using the asymptotic expansion of $\Gamma(p,x)$ for large arguments x [19], one can obtain

$$C_\nu=-\frac{Q}{3b}a^{\nu-2}\exp(-ba^3)\Bigg|_{a_{\min}}^{a_{\max}}=-\frac{ak^2v_T^2(a)}{3\omega^2}Qa^\nu\times\exp\left[-\frac{\omega^2}{k^2v_T^2(a)}\right]\Bigg|_{a_{\min}}^{a_{\max}}. \tag{27}$$

In this approximation expressions (22)–(24) become very simple, namely,

$$C_p=\omega_p^2(a)\mathcal{E}(a)\big|_{a_{\min}}^{a_{\max}}, \tag{28}$$

$$C_J=\omega_J^2(a)\mathcal{E}(a)\big|_{a_{\min}}^{a_{\max}}, \tag{29}$$

$$C_{pJ}=\omega_p(a)\omega_J(a)\mathcal{E}(a)\big|_{a_{\min}}^{a_{\max}}, \tag{30}$$

where $\omega_p^2(a)=N_0q^2(a)/\varepsilon_0m(a)$, $\omega_J^2(a)=4\pi GN_0m(a)$, and $\mathcal{E}(a)$ is shorthand for

$$\mathcal{E}(a)=-\frac{2\sqrt{\pi}}{3\omega^2kv_T(a)}\left(\frac{a_0}{a}\right)^{\mu-1}\exp\left(-\frac{\omega^2}{k^2v_T^2(a)}\right). \tag{31}$$

Using the same notations yields

$$\Omega_p^2 = -\frac{1}{\mu}\left(\frac{a_0}{a}\right)^{\mu-1}\omega_p^2(a)\Bigg|_{a_{\min}}^{a_{\max}}, \tag{32}$$

$$\Omega_J^2 = -\frac{1}{\mu-4}\left(\frac{a_0}{a}\right)^{\mu-1}\omega_J^2(a)\Bigg|_{a_{\min}}^{a_{\max}}, \tag{33}$$

$$\Omega_{pJ}^2 = -\frac{1}{\mu-2}\left(\frac{a_0}{a}\right)^{\mu-1}\omega_p(a)\omega_J(a)\Bigg|_{a_{\min}}^{a_{\max}}, \tag{34}$$

and also

$$\Delta_p = -\frac{3k^2}{2\Omega_p^2}\left(\frac{a_0}{a}\right)^{\mu-1}\frac{\omega_p^2(a)v_T^2(a)}{\mu+3}\Bigg|_{a_{\min}}^{a_{\max}}, \tag{35}$$

$$\Delta_J = -\frac{3k^2}{2\Omega_p^2}\left(\frac{a_0}{a}\right)^{\mu-1}\frac{\omega_J^2(a)v_T^2(a)}{\mu-1}\Bigg|_{a_{\min}}^{a_{\max}}, \tag{36}$$

$$\Delta_{pJ} = -\frac{3k^2}{2\Omega_p^2}\left(\frac{a_0}{a}\right)^{\mu-1}\frac{\omega_p(a)\omega_J(a)v_T^2(a)}{\mu+1}\Bigg|_{a_{\min}}^{a_{\max}}. \tag{37}$$

Now we are ready to look at peculiarities of dust-acoustic modes in dusty and self-gravitating plasmas with different μ. Since both $\varepsilon(\omega,k)$ and ω are now complex, $\varepsilon(\omega,k)=\varepsilon_r+i\varepsilon_i$ and $\omega=\omega_r+i\gamma$, we will make the usual assumption that $|\omega_r|\gg|\gamma|$ and $|\varepsilon_r|\gg|\varepsilon_i|$.

We first of all consider a dusty plasma without self-gravitation ($\Omega_J=0=\Omega_{pJ}$), and hence the coefficients Δ_J, Δ_{pJ}, C_J, and C_{pJ} are equal to zero. Then the dispersion relation for dust-acoustic modes in a plasma with a dust size distribution is recovered [3], in the case when constant dust charges are assumed. Numerical computations [3] seem to indicate that the power-law distribution of dust particle sizes leads to a negligibly small effect on the Landau damping, when the upper (a_{max}) and lower (a_{min}) dust sizes are adjusted so that the average particle sizes are the same as assumed in a monosized distribution. We consider a size spectrum (9), when $a_{\max}\gg a_{\min}$ and $\mu>1$, and can then give some analytical expressions. The dispersion relation (15) thus becomes

$$1+\frac{1}{k^2\lambda_D^2}\left[1+\frac{i\sqrt{\pi}\omega}{k(1+\delta)}\left(\frac{1}{v_{Ti}}+\frac{\delta}{v_{Te}}\right)\right] -\frac{\Omega_p^2}{\omega^2}\left(1+\frac{3\mu k^2v_T^2(a_{min})}{2(\mu+3)\omega^2}\right)+\frac{2i\sqrt{\pi}\mu\Omega_p^2}{3\omega k v_T(a_{min})} \times\exp\left(-\frac{\omega^2}{k^2v_T^2(a_{min})}\right)=0, \tag{38}$$

with $\lambda_D^{-2}=\lambda_{De}^{-2}+\lambda_{Di}^{-2}$ and $\delta=\lambda_{Di}^2/\lambda_{De}^2$. Note that the dispersion law is insensitive to the upper limit of integration as long as $a_{\max}\gg a_{\min}$. It is easily seen that when $k^2\lambda_D^2\ll 1$, the frequency and damping rate of the dust-acoustic mode are given by

$$\omega_r^2=\frac{1}{2}k^2U_{da}^2(1-k^2\lambda_D^2)\left[1+\sqrt{1+\frac{12\mu(1+k^2\lambda_D^2)}{(\mu+3)\beta}}\right], \tag{39}$$

and, if $\beta\gg 12\mu/(\mu+3)$,

$$\gamma\simeq-\sqrt{\frac{\pi}{8}}kU_{da}\left\{(1+\delta)^{-3/2}\left(\frac{\Omega_p}{\omega_{pi}}+\frac{\Omega_p}{\omega_{pe}}\delta^{3/2}\right) +\frac{2\mu}{3}\sqrt{\beta}\exp\left[-\frac{\beta}{4}\left(1+\sqrt{1+\frac{12\mu}{(\mu+3)\beta}}\right)\right]\right\}, \tag{40}$$

where the dust-acoustic velocity $U_{da}=\Omega_p\lambda_D$ has been introduced and β is defined as $\beta=\lambda_D^2/\lambda_{D1}^2$ with $\lambda_{D1}^2=v_T^2(a_{min})/2\Omega_p^2$. Obviously $\Omega_p^2=(\mu-1)\omega_p^2(a_{\min})/\mu$ since $\mu>1$ and $a_{min}\ll a_{max}$.

Generally speaking, these equations reveal an effect of the dust size distribution on the real frequency and damping rate of the dust-acoustic waves. Just to show this, we compare Eqs. (39) and (40) to corresponding values when all the dust particle sizes are $a=a_{\min}$ and the total density equals N_0. Using the equation of charge neutrality (13), we can easily compute the ratios of two real frequencies or two damping rates for both situations, with and without size distribution. Without going into details, it can be shown that both ratios are approximately characterized by the same small dependence on μ, namely, $\sim(\mu-2)/\mu$. However, the relative damping rate γ/ω_r for dust-acoustic modes in dusty plasmas with a power-law dust size distribution is almost independent of $n_d(a)$, similar to what was obtained in [3].

Returning now to the general case of self-gravitating plasmas, it is easy to verify that the structure of Eq. (15) becomes even more dependent on μ because of the gravitational (Ω_J, Δ_J) and hybrid terms (Ω_{pJ}, Δ_{pJ}). Of course, we suppose that μ does not equal one of the critical values occuring in the expressions (16)–(21), but these can easily be dealt with by taking the appropriate limiting expressions. Whereas the plasma terms (16) and (19) are always determined by the smallest particles ($a_{\min}$), an effective Jeans frequency (17) and the hybrid frequency is determined by either the smallest or largest grains, depending on the magnitude of μ. Since for $a_{min}\ll a_{max}$

$$\Omega_p^2=\frac{\mu-1}{\mu}\omega_p^2(a_{\min})\left[1-\left(\frac{a_{min}}{a_{max}}\right)^{\mu}\right], \tag{41}$$

$$\Omega_J^2=\frac{\mu-1}{\mu-4}\omega_J^2(a_{\min})\left[1-\left(\frac{a_{min}}{a_{max}}\right)^{\mu-4}\right], \tag{42}$$

$$\Omega_{pJ}^2=\frac{\mu-1}{\mu-2}\omega_p(a_{min})\omega_J(a_{min})\left[1-\left(\frac{a_{min}}{a_{max}}\right)^{\mu-2}\right], \tag{43}$$

it turns out that if $\mu>4$, the results for Ω_J and Ω_{pJ} are weighted towards the smaller sized particles, which are more abundant. And vice versa, the upper limit ($a_{\max}$) becomes dominant for an effective Jeans frequency when $\mu<4$, and for the hybrid term when $\mu<2$. Bearing in mind the observational data concerning dust distributions in real dusty plasma objects, the cases $\mu>4$ and $2<\mu<4$ are of a special interest.

We first deal with the case $\mu>4$ and consider the real part of the general dispersion equation (15), neglecting for simplicity the small thermal terms. In the dust-acoustic regime, this reduces to

$$\omega^4-\omega^2(k^2U_{da}^2-\Omega_J^2)-Ck^2U_{da}^2\Omega_J^2=0, \tag{44}$$

where $C=4/(\mu-2)^2$. As the product of the roots of Eq. (44) for ω^2 is negative, there is always one positive solution for ω^2, corresponding to a stable mode, as well as a purely imaginary mode for which $\omega^2<0$.

In the case of a self-gravitating plasma when a dust-acoustic mode occurs ($k^2U_{da}^2 \geqslant \Omega_J^2$), the roots are given by

$$\omega_1^2 \simeq k^2U_{da}^2-\Omega_J^2(1-C) \tag{45}$$

and

$$\omega_2^2 \simeq -C\Omega_J^2. \tag{46}$$

The first solution gives a generalized dust-acoustic wave. The dust distribution tends to reduce the effective Jeans frequency and thereby increase the phase velocity of this mode. When the thermal effects are incorporated into the analysis, one finds a slight correction for the dust-acoustic branch (45), so that

$$\omega_1^2 \simeq k^2U_{da}^2\left(1+\frac{3\mu}{(3+\mu)\beta}\right)-\Omega_J^2(1-C). \tag{47}$$

It is readily verified that this mode is always damped. We do not give the details, because this becomes very repetitive, but one can obtain the damping rate as

$$\gamma=-\sqrt{\frac{\pi}{8}}kU_{da}\left\{(1+\delta)^{-3/2}\left(\frac{\Omega_p}{\omega_{pi}}+\frac{\Omega_p}{\omega_{pe}}\delta^{3/2}\right)+\frac{2\mu\sqrt{\beta}}{3}\exp\left[-\frac{1}{2}\left(\frac{3\mu}{\mu+3}+\beta\right)+\frac{\beta\Omega_J^2(1-C)}{2k^2U_{da}^2}\right]\right\}. \tag{48}$$

Although these relations bear a resemblance to the damping rate of the dust-acoustic wave (40), one can see the influence of self-gravitational effects on the growth of the damping rate if the exponent in the size distribution increases. Only in the special case when dealing with very large values of β, the first term between brackets in Eq. (48) dominates and thus, the effect of self-gravitation on the damping rate diminishes, and the latter reduces to the monosized dust case (40).

Hence, it is more interesting when we restrict our analysis to those values of β for which the second term of Eq. (48) prevails. With regard to possible parameters of self-gravitating plasmas, it is quite realistic to assume that β remains smaller than 50 [13]. Then the damping rate is modified by self-gravitational effects. The latter can lead to a considerable growth of the damping of the dust-acoustic mode in a monosized plasma [13]. Now we examine the influence of the size distribution on the damping rate of the dust-acoustic mode. The self-gravitational terms in Eq. (48) are proportional to

$$\frac{\Omega_J^2(1-C)}{k^2U_{da}^2}=\frac{\omega_J^2(a_{\min})\mu}{k^2\lambda_D^2\omega_p^2(a_{\min})(\mu-4)}\left[1-\frac{4}{(\mu-2)^2}\right]. \tag{49}$$

This reveals the effect on γ of different μ in the size distribution power law. From the last expression, it is quite obvious that for values of $\mu\sim5$–7 the contribution of this term is rather enhanced due to the size distribution. Only for very large $\mu>10$ does the dependence (49) tend to 1, and the influence of the dust distribution, consequently, is of little importance.

The negative root (46) gives rise to a weakly unstable mode, the frequency of which is completely defined by μ in the dust size distribution. This mode is hence a dust distribution instability. This repeats the results of Meuris *et al.* [6], because if one replaces the double summation in their equation (32) by a double integral this matches our equation (44). Following the notations of Meuris *et al.* [6] $k^2U_{da}^2>\Omega_J^2$ corresponds to their condition $A>B$.

Next we consider the opposite case, when $k^2U_{da}^2 \leqslant \Omega_J^2$. In this limit, the self-gravitational effects dominate and the roots of (44) are of the form

$$\omega_1^2 \simeq -\Omega_J^2+k^2U_{da}^2(1-C) \tag{50}$$

and

$$\omega_2^2 = Ck^2U_{da}^2. \tag{51}$$

The mode with $\omega^2<0$ gives a modified Jeans instability, and a dust size distribution tends to increase its growth rate, whereas the solution $\omega^2>0$ is a stable mode, which does not exist in the case of monosized dust distribution. This mode is always damped with a decrement

$$\gamma=-\sqrt{\frac{\pi}{8}}kU_{da}\left\{(1+\delta)^{-3/2}\left(\frac{\Omega_p}{\omega_{pi}}+\frac{\Omega_p}{\omega_{pe}}\delta^{3/2}\right)+\frac{2\mu\sqrt{\beta}}{3}\exp\left[-\frac{C\beta}{2}\right]\right\}. \tag{52}$$

Qualitatively similar results are obtained by Meuris *et al.* [6] in the fluid description, when the dust-acoustic waves are treated in a plasma with a number of discrete charged and neutral dust species. However, the damping effect for different modes is not included in the fluid approach [6].

Finally, the discussion about dust-acoustic modes in self-gravitating plasmas can be rounded off by looking at the case when the exponent μ in the size distribution is sufficiently small, i.e., $2<\mu<4$, and the heavier particles determine the gravitational terms in the general dispersion law (15). In this limit the dispersion relation (44) is still valid but the values of Ω_J^2 and C change to

$$\Omega_J^2 = \frac{\mu-1}{4-\mu}\omega_J^2(a_{\min})\left(\frac{a_{max}}{a_{min}}\right)^{4-\mu}, \tag{53}$$

$$C = 1 - \frac{\mu(4-\mu)}{(\mu-2)^2}\left(\frac{a_{min}}{a_{max}}\right)^{4-\mu} \simeq 1. \tag{54}$$

There is now an almost complete factorization of Eq. (44), viz.,

$$(\omega^2 - k^2 U_{da}^2)(\omega^2 + \Omega_J^2) \simeq 0, \tag{55}$$

and the plasma and gravitational disturbances decouple and can develop almost independently.

Thus, the remarkable property of self-gravitating plasmas is that the results are strongly dependent on the exponent in the size distribution of the dust particles.

IV. RESULTS

In many astrophysical dusty plasmas, the size of the dust components spans a wide range. For this reason, we generalized the kinetic approach to low-frequency modes in self-gravitating plasmas with a size distribution. Following observational results, we modeled a size distribution as a descending power law in a range of particle sizes, as observed in various heliospheric plasmas. For planetary rings the self-gravitational effects are usually much less important than the gravitational attraction of the planet itself. Unfortunately, not enough is reliably known about dust size and mass distributions outside the solar system to give quantitative comparisons for potentially interesting configurations like molecular clouds.

The general kinetic dispersion law for electrostatic modes, which is quite complicated, was adapted to inertialess electrons and ions. We then discussed several, practically, important size distributions. A detailed study of the longitudinal waves has been made in the dust-acoustic regime, including the plasma and gravitational modes, their damping and growth rates.

Although some of the results, like the dust distribution instability and a stable mode due to the size spectrum of the dust particles, corroborate what was obtained in the fluid model [6], the inclusion of a size distribution into a kinetic analysis demonstrates a novel property of such media. Whereas the effective plasma terms are always weighted towards the smallest particles, which are more abundant, the effective Jeans and hybrid terms can be determined by either the smallest or the largest grains, depending on the precise slope of the power-law distribution. As a result, the coupling between the plasma and gravitational waves can be strongly dependent on the parameters of the power-law distribution.

We investigated damping effects for analogues of dust-acoustic modes, to show whether a power-law distribution of dust-particle sizes influences the Landau damping or not, and found that for certain parameter values there can be a considerable damping, completely specified by self-gravitation. This effect never occurs in usual dusty plasmas.

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